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CONTROL OF SENSORS FOR SEQUENTIAL DETECTION A STOCHASTIC APPROACH

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The project was aimed at developing computationally efficient rigorous design methods for control of sensing resources for adaptive information gathering in sequential detection and classification problems. The proposed approach centered on obtaining optimal stochastic sensor management policies. Sequential tests for detection and classification were considered, that have as cost function the expected number of measurements required to guarantee the probability of error to be below specified limits, while explicitly including constraints on sensor usage. The proposed research has been successfully completed.

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Final Performance Report for Project AFOSR Award FA9550-10-1-0348 titled 'CONTROL OF SENSORS FOR SEQUENTIAL DETECTION A STOCHASTIC APPROACH'

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1. *Project objectives:* The project was aimed at developing computationally efficient rigorous design methods for control of sensing resources for adaptive information gathering in sequential detection and classification problems. Developing and maintaining situation awareness in a complex, dynamic, and information rich world requires efficient methods for information gathering, interpretation, management, and fusion. This proposal concentrated on the following fundamental problem in this domain: "How do we obtain scalable methods with analytical performance guarantees for controlling sensing resources to gather the most relevant and useful data for sequential detection and classification, to decide on one among many hypotheses with minimal observation cost?" Detection and classification problems arise in multiple contexts of interest to the Air Force, including intruder detection, target search including unexploded ordinance and landmine detection, target tracking, threat classification, and so on. Most of the existing solutions for sensor management for either estimation or detection involve setting up the sensor usage optimization problem as a partially observed Markov decision process (POMDP). However, solution of such problems becomes computationally intractable as either the number of sensors or the problem horizon increases.

   The proposed approach centered on obtaining optimal stochastic sensor management policies. Sequential tests for detection and classification were considered, that have as cost function the expected number of measurements required to guarantee the probability of error to be below specified limits, while explicitly including constraints on sensor usage. Rather than heuristic solutions that aim at decreasing some cost function myopically, this proposal aimed at developing a fundamentally new approach and extend the sequential probability ratio test for the case when sensors are selected in a probabilistic fashion. The first step was to establish the convergence, optimality, and performance of the test in this context. Then, optimization problems to design the sensor selection probabilities and provide guaranteed performance were sought to be posed and solved through efficient algorithms.

2. *Outcomes:* The proposed research has been successfully completed. We provide below a synopsis of the research done as part of the project and provide references to publications were further information can be found.

   Research activities and findings: The first step in the project was to establish that the classical sequential probability ratio test (SPRT) was optimal even when sensors were selected stochastically and constraints on sensor usages were present. Although some proofs in the

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literature were available for the case when the number of measurements is very large, such asymptotic proofs do not hold when there are constraints on sensor usage present. For this case, we obtained a dynamic programming based proof that rigorously demonstrates that SPRT remains the optimal test even if sensors are chosen stochastically. We posed the problem as that of minimizing a cost function that has two terms - a Bayesian loss function and the sum of usage costs for the sensors - by selecting one sensor out of multiple ones present at every time step. As in the classical SPRT, the optimal decision rule is given by a Bayesian test on the collected measurements. The specific sensor selection strategy does not impact this decision rule. The stopping rule can be posed in terms of a likelihood ratio. The calculation of the likelihood ratio depends on the specific sensor selection strategy. Finally, we proved that the Wald-Wolfowitz theorem also holds in this case, implying that no other test can achieve the desired probability of error with lesser expected observation cost. These results are fully described in the conference paper:

- V. Katewa and V. Gupta, "On the Optimality of Sequential Tests with Multiple Sensors," in 2012 American Control Conference, Montreal, Canada, June 2012.

Having established the optimality of SPRT, the next step was to identify the optimal probabilities with which sensors must be chosen. We began with identifying off-line rules for obtaining such probabilities. The optimization problem in this case is a sum-of-ratios Linear Fractional Program (LFP). Even though we proved that for our problem the feasible region is convex, an arbitrary su-of-ratios LFP even over a convex set is known to be computationally hard to solve in general. However, using the structure of our problem, we identified several cases in which the optimal probability vector could be identified with complexity that was only linear in the number of sensors. As an example, we proved that if the sensors can be ordered in a certain sense related to the Kullback-Leibler divergence, then a greedy algorithm is optimal. Such a case can model many examples of practical interest, such as Amplitude Detection in additive white Gaussian noise, or Energy Detection with uniform observation costs. For this case, we could prove that the optimal solution is such that there is at most one sensor that is used with a probability greater than zero, such that its usage constraint is not satisfied with equality. All other sensors are either used with probability equal to zero, or with a probability such that its usage constraint is satisfied with equality. Thus, a greedy search algorithm can be designed to obtain the optimal probability of sensor usage with complexity only linear in the number of sensors. Moreover, for more general cases, we provided sub-optimal algorithms that had only quadratic computational complexity in the number of sensors. This sub-optimal algorithm was based on a result that showed that a result on sensor image similar to the one mentioned above for the case of orderable sensors holds more generally with the following difference: instead of at most *one* sensor that can have a selection probability different from both zero and its usage constraint, now there can be at most *two* such sensors. Thus, if we knew which two sensors were in this group and which sensors were fully used, then the optimal probability vector can be obtained by solving an optimization problem with three variables, irrespective of the total number of sensors. The source of computational complexity of the problem lies precisely in identifying the identity of these sensors. In the orderable case this identification was done by moving through the sensors one at a time. In the general case, two sensors may be simultaneously active and this procedure breaks down. Nevertheless, we provided a sub-optimal algorithm that has complexity quadratic in the number of sensors. This sub-optimal algorithm is a generalization of the optimal algorithm from the orderable case and yields the optimal solution in special cases, e.g., if there are no constraints on sensor usage

or if the sensors are orderable. Moreover, in other cases, the algorithm yields a solution that is close to the one obtained using a (computationally expensive) optimal algorithm. Finally, the design of the sensor selection probability was based on using Wald's approximation on the expected number of measurements. Since we are interested in the case when the number of measurements need not be infinite, we provided an upper bound on the error incurred on the actual sensor usage due to this approximation. This bound can be used to set a safety margin on the constraints on sensor usage.

These results were described in two papers:

- C.-Z. Bai, V. Gupta, and Y.-F. Huang, "Sequential Hypothesis Testing with Off-line Randomized Sensor Selection Strategy," in 2012 IEEE International Conference on Acoustics, Speech, and Signal Processing, Kyoto, Japan, March 2012.
- C.-Z. Bai, V. Katewa, V. Gupta, and Y.-F. Huang, "A Stochastic Sensor Selection Scheme for Sequential Hypothesis Testing with Multiple Sensors," in IEEE Transactions on Signal Processing, Submitted, 2012.

The final step was to consider on-line sensor selection rules, i.e., strategies in which the sensor that is selected can depend on the measurements taken till that time. Such on-line strategies are considerably more difficult to analyze and design. Once again, we introduced an observation cost associated with every individual sensor, and aim to design a sensor selection strategy that minimizes the expected total observation cost. Using a dynamic programming interpretation, we imposed a strong sense of optimality in which the cost-to-go is minimized at every time step (and not merely asymptotically in the number of measurements). We characterize several properties of the optimal sensor selection strategy, for instance, showing that the optimal strategy is stationary. Thus, the optimal strategy for determining the probabilities of sensor usage at any time is time-invariant, while the probabilities themselves may be varying with time. We showed that the problem of finding the optimal sensor selection strategy was akin to solving the renewal equation that is known to be computationally hard in probability theory. In order to reduce the computational effort, we propose a novel numerical algorithm in which we partitioned the state space into three regions and solve for the optimal strategy in each region. Numerical results show that the resulting strategy was very close to the optimal strategy in almost every case.

These results was described in the paper:

- C.-Z. Bai, V. Gupta, and Y.-F. Huang, "An On-line Sensor Selection Algorithm for SPRT with Multiple Sensors," in IEEE Transactions on Automatic Control, To be Submitted, 2013.

In summary, by performing core research on information gathering and fusion, this proposal has conducted research that is directly relevant to control and coordination of sensors in uncertain, sensor rich, dynamically changing and networked environments, and is essential for the new heavily automated paradigms of operation that the US Air Force is interested in. Existing approaches for sensor management are based on setting up the sensor usage optimization problem as a partially observed Markov decision process. Since such problems become computationally intractable as the number of sensors or the problem horizon increase, many interesting heuristic approximations based on myopic decrease in some cost measure have been proposed. However, techniques with analytic guarantees of performance for sequential detection problems are sorely needed. Through a new stochastic approach to sensor selection,

this work has developed computationally efficient methods to gather only the most important information relevant for detection and classification, thus addressing a key challenge in ensuring situational awareness in sensor rich, complex and heavily automated situations.

Conference and Journal Papers: The following conference and journal papers have been submitted / presented about the work carried out so far as part of the project:

- V. Katewa and V. Gupta, "On the Optimality of Sequential Tests with Multiple Sensors," in 2012 American Control Conference, Montreal, Canada, June 2012.
- C.-Z. Bai, V. Gupta, and Y.-F. Huang, "Sequential Hypothesis Testing with Off-line Randomized Sensor Selection Strategy," in 2012 IEEE International Conference on Acoustics, Speech, and Signal Processing, Kyoto, Japan, March 2012.
- C.-Z. Bai, V. Katewa, V. Gupta, and Y.-F. Huang, "A Stochastic Sensor Selection Scheme for Sequential Hypothesis Testing with Multiple Sensors," in IEEE Transactions on Signal Processing, Submitted, 2012.
- C.-Z. Bai, V. Gupta, and Y.-F. Huang, "An On-line Sensor Selection Algorithm for SPRT with Multiple Sensors," in IEEE Transactions on Automatic Control, To be Submitted, 2013.

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